

# APPLICATION OF A TWO-DIMENSIONAL PHYSICAL OPTICS MODEL IN THE ANALYSIS OF MICROSTRIP ANTENNAS ON A FINITE GROUND PLANE

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**Abstract** - A two-dimensional (2D) physical optics model for a magnetic line source located near the edge of a semi-infinite structure is applied to investigate the radiation characteristics of a microstrip patch antenna located on a finite ground plane. The 2D model is coupled with the 3D model using a relationship between the incident field at the edge of the finite ground plane and the amplitude of the magnetic line source. The incident field at the edge is expressed in terms of so-called expansion waves. Our formulation to take the diffraction effects into consideration is simple and not time-consuming because it involves only analytical expressions. The experimental results are in good agreement with the proposed model.

## 1. Introduction

The design of a microstrip antenna is normally based on the assumption that a ground plane has infinite dimensions. In a real situation the ground plane always has finite dimensions. The finite size of the ground plane has a noticeable impact on the radiation pattern. A rigorous way to take these effects into consideration leads to integral equation formulations [1-4]. They are potentially very rigorous but their implementation requires a lot of computer resources because the number of unknowns is proportional to the total size of the ground plane. A simplified approach involves approximations that are based on a physical insight of the diffraction phenomena. A good approximation can be obtained by using a physical optics (PO) model [5-10]. The PO model replaces the finite layer structure by equivalent volume and surface polarization electric currents. These currents are the same as in the infinite structure. The PO model gives a good estimation for a radiation pattern. Although the main idea of this PO method is simple, its application is not because the PO currents are not known analytically. This problem can be solved in several ways: discrete approximation [5] or asymptotic approximation [6-10] of the PO field. There are also two possible ways of expressing fields generated by the PO currents. We can express fields in terms of the PO currents [5-8] as shown in Fig. 1a or we can also express fields in terms of the truncated PO current [9,10] as shown in Fig. 1b.

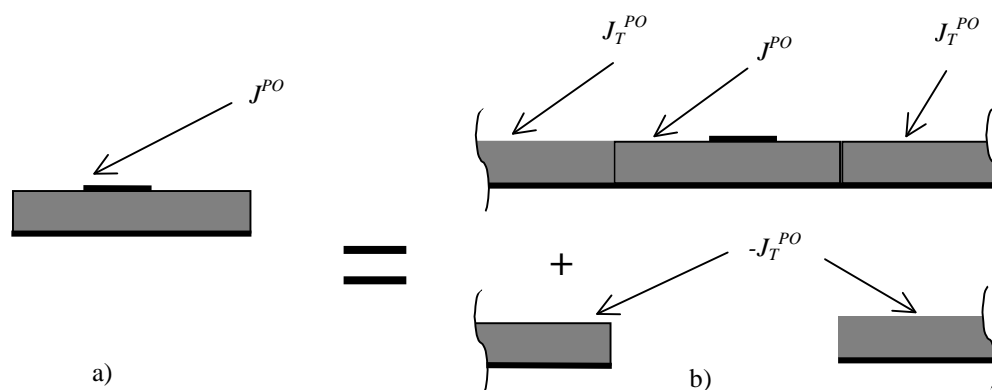


Fig.1 Two formulations for the PO model

For the second PO formulation shown in Fig. 1b the total radiation pattern is written as a sum of two terms: pattern over the infinite structure and the diffraction term caused by the truncation of the structure. This modified formulation requires minor modification of existing models for the infinite structure. The influence of the finite ground plane is taken into consideration by an additional diffraction term. We have applied this formulation to model line sources [9] and microstrip antennas [10]. In the 3D case [10] the diffraction term is expressed in terms of an

integral over the perimeter of the ground plane. This integral is calculated numerically. As a consequence the calculation time becomes proportional to a size of the ground plane. In the present paper we suggest to combine the 2D PO model with the 3D PO model in order to avoid numerical calculation of the integral. This step is similar to asymptotic evaluation of the integral. A comparison with experimental results shows the validity of our approach.

## 2. Theory

Let's consider a semi-infinite dielectric structure excited by an arbitrary 3D source (microstrip antenna)  $J(x,y,z)$  and an arbitrary 2D line source located at a distance  $d$  from the edge. The geometry is shown in Fig. 2.

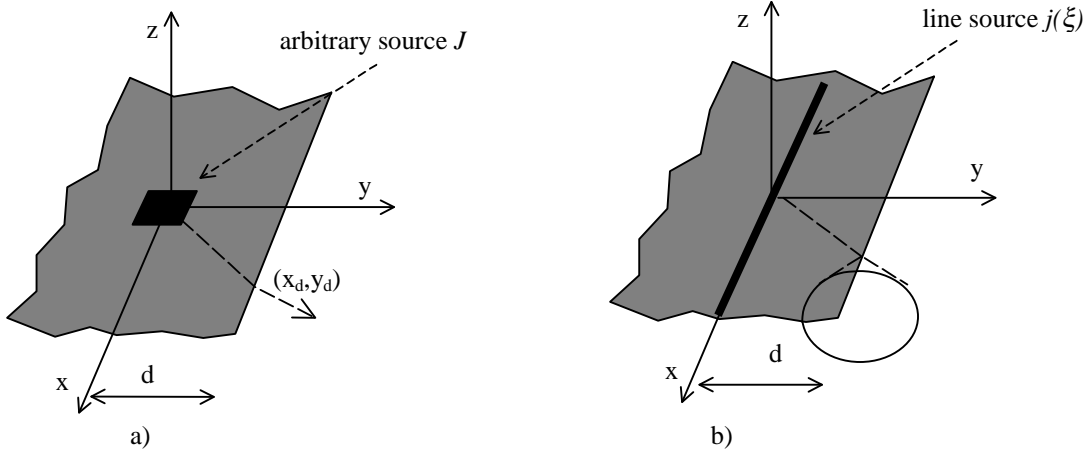


Fig. 2 Geometry of a semi-infinite dielectric structure.

We assume that the 3D source is known. The current flowing on the microstrip antenna is the same as for an infinite structure. By using the Fourier transform in the coordinate  $x$  parallel to the edge we obtain a 2D problem in the spectral domain. The original source is replaced now by line sources parallel to the edge.

$$j(\xi, y, z) = \int_{-\infty}^{+\infty} J(x, y, z) e^{+j\xi x} dx \quad (1)$$

For each value of the spectral parameter  $\xi$  a solution can be obtained using the 2D PO model suggested in [9]. In the far zone the field can be written in the following form

$$f_x(\xi, \rho, \phi) = D_f(\phi, \xi) \frac{e^{-jk_0 s}}{\sqrt{s}} j(\xi) = D_f(\phi, \xi) \frac{e^{-jk_\xi \rho - j\xi x}}{\sqrt{s}} j(\xi) \quad (2)$$

where  $s$  is a ray fixed coordinate,  $\rho = \sqrt{y^2 + z^2}$ ,  $\phi = \arctg\left[\frac{y}{z}\right]$ ,  $k_\xi = \sqrt{k_0^2 - \xi^2}$ ,  $k_0 = \frac{2\pi}{\lambda}$ , and  $D_f$  is a diffraction coefficient calculated analytically [9]. In order to return to the spatial domain we have to perform the inverse Fourier transform

$$F_x(r, \theta, \varphi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f_x(\xi, \rho) e^{-j\xi x} d\xi \quad (3)$$

This integral can be estimated using the saddle point asymptotic method. In the vicinity of the saddle points  $\xi = \xi_0$  the functions  $D_f(\xi)$  and  $j(\xi)$  are smooth and they are replaced by their values in the saddle point for all spectral parameters.

$$F_x(r, \theta, \varphi) = \sqrt{\frac{k_0}{2\pi}} D_f(\xi_0) j(\xi_0) \cos \alpha \frac{1}{r} e^{-jk_0 r + j\frac{\pi}{4}} \quad (4)$$

where  $r = s$ ,  $\alpha = 90 - \beta$  and  $\beta$  is the angle of the diffraction cone in Fig. 2b. The only term in (4), which is unknown for the moment is the spectral current  $j(\xi)$ . We can use directly (1) in order to calculate the spectral components but this

method is not very attractive because we have to keep information about the primary current  $J$ . Instead, we express the spectral current  $j(\xi)$  in terms of the amplitude of the incident field at the point of diffraction  $Q(x_d, y_d=0)$  at the edge. The incident field can be expressed in terms of so-called expansion waves [11]. Each expansion wave gives a contribution to the incident field at the edge. We consider an arbitrary expansion wave because the transformation is similar for all expansion waves. In order to establish the necessary relationship let's consider an infinite dielectric structure excited by a source  $J$  located at the point  $(x_s=0, y_s=-d)$ . The incident field can be expressed in terms of the Fourier Transform for the 2D problem

$$F_x^{inc}(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f^{inc}(\xi) j(\xi) e^{-j\xi x} d\xi \quad (5)$$

Using again the method of the saddle points we can estimate (5)

$$F_x^{inc}(x, y) \approx \sqrt{\frac{P}{2\pi}} f^{inc}(\xi_0) j(\xi_0) \cos\varphi \frac{1}{\sqrt{\rho_s}} e^{-jPR + j\frac{\pi}{4}} \quad (6)$$

where  $P$  is the propagation constant of the expansion wave,  $\rho_s = \sqrt{(x_d^2 - x_s^2) + (y_d^2 - y_s^2)} = \sqrt{x_s^2 + d^2}$  and

$\cos\varphi = \frac{d}{\rho_s}$ . From (6) it follows

$$j(\xi_0) \approx \sqrt{\frac{2\pi}{P}} \frac{\sqrt{\rho_s}}{\cos\varphi} \frac{F_x^{inc}}{f_T^{inc}} e^{-j\frac{\pi}{4}} \quad (7)$$

where  $f_T^{inc} = f(\xi_0) e^{-jPR}$  is a solution of the 2D problem. After the substitution of (7) into (4) we obtain an analytical expression for the diffraction term in the far field

$$F_x(r, \theta, \varphi) = \sqrt{\frac{k_0}{P}} \frac{\cos\alpha}{\cos\varphi} \sqrt{\rho_s} D_f(\xi_0) \frac{F_x^{inc}}{f_T^{inc}} \frac{1}{r} e^{-jk_0 r} \quad (8)$$

This expression can be rewritten in a more familiar form, which is used in the geometrical theory of diffraction, by introducing a virtual radius  $\rho_M$  of the diffracted field

$$\rho_M = \frac{k_0}{P} \frac{\cos^2\alpha}{\cos^2\varphi} \rho_s \quad (9)$$

The final expression for the contribution from an arbitrary expansion wave to the diffraction term in the far field is

$$F_x(r, \theta, \varphi) = \sqrt{\rho_M} D_f(\xi_0) \frac{F_x^{inc}}{f_T^{inc}} \frac{1}{r} e^{-jk_0 r} \quad (10)$$

The total diffraction term includes contributions from all expansion waves.

### 3. Numerical Results

The formulation developed in the previous section allows to investigate the radiation pattern of a rectangular patch antenna located on a finite sized ground plane. As an example we have chosen a structure described in [12]. The dimensions of the rectangular probe-fed patch antenna are 54mm×38mm, substrate thickness 3.175mm, dielectric constant  $\epsilon_r=2.55$  and the frequency 2.35GHz. The size of the ground plane is 267mm×355mm. The far field pattern in the E plane is shown in Fig. 3. The agreement between our model (plotted by a solid line) and experimental results from [12] (plotted by delta signs) is very good.

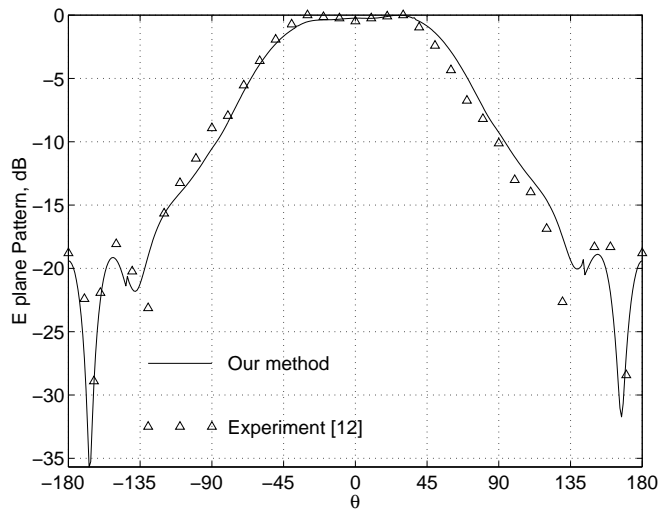


Fig. 3 Radiation pattern of a rectangular microstrip antenna located on a finite ground plane.

#### 4. Conclusion

The 2D PO model was combined with the 3D PO model in order to analyze a microstrip antenna on a finite ground plane. The proposed approach is fast because the algorithm is based on analytical expressions. The calculation time does not depend on the size of the ground plane and it is determined by the time that it is necessary to calculate the microstrip antenna on an infinite ground plane. Although the PO model is based on an approximation, it gives an estimate that is acceptable in practical situations. The proposed model can be used in microstrip antenna design.

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